Find the linearization of $f(x) = 2 + \int_0^x \frac{10}{1+z}\,dz$ at $x=0$

pt. $(0, 2)$

$L(x) = \overset{\text{pt}}{f(a)} + \underbrace{f'(a)}_{\text{slope}}(x - \overset{\text{pt}}{a})$

Slope: 10

$\frac{d}{dx}2 + \frac{d}{dx}\int_0^x \frac{10}{1+z}\,dz$

$\frac{d}{dx}2 \to 0$

$\frac{d}{dx}\int_0^x \frac{10}{1+z}\,dz \to \frac{10}{1+x} = \frac{10}{1+0} = 10$

$$\boxed{L(x) = 2 + 10(x)}$$

Weekly Review #7

1/19/10

1. (a) $\lim_{x \to 1^+} \begin{cases} x^2 - 2, & x < 1 \\ -\frac{1}{2}x + 1, & x \geq 1 \end{cases}$ use $-\frac{1}{2}x+1 \to -\frac{1}{2}(1)+1 = \boxed{+\frac{1}{2}}$

(b) $\lim_{x \to \infty} \frac{6x+1}{|6-2x|}$ as $x \to \infty$ the constants disappear in significance and both top and bottom are positive

so $\frac{6x}{2x} \Rightarrow \boxed{3}$

(c) $\lim_{x \to 5} \frac{5-6x+x^2}{5-x} \Rightarrow \lim_{x \to 5} \frac{(x-5)(x-1)}{5-x} \Rightarrow -\frac{\cancel{(x-5)}(x-1)}{\cancel{(x-5)}} \Rightarrow 1-x \Rightarrow 1-5$

$\boxed{=-4}$

(d) $\lim_{x \to 3} 5-2x+x^2 \Rightarrow 5-2(3)+(3)^2 \Rightarrow \boxed{8}$

2 a) $f(x) = x\csc(x)$ at $x = \frac{\pi}{6}$

$f'(x) = x \cdot -\csc(x)\cot(x) + \csc(x) \Rightarrow \csc(x)\left(1 - x\cot(x)\right)$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\csc\left(\frac{\pi}{6}\right) = 2$

$f'\left(\frac{\pi}{6}\right) = \csc\left(\frac{\pi}{6}\right)\left(1 - \frac{\pi}{6}\left(\cot\left(\frac{\pi}{6}\right)\right)\right) \Rightarrow 2\left(1 - \frac{\pi}{6}(\sqrt{3})\right) = \boxed{2 - \frac{\pi\sqrt{3}}{3}}$ slope

orig. pt. $f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}\csc\left(\frac{\pi}{6}\right) \Rightarrow \frac{\pi}{3}$ $\quad \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ ≈ 0.186

2pts tangent $\Rightarrow$ $\boxed{L(x) = 2 - \frac{\pi\sqrt{3}}{3}\left(x - \frac{\pi}{6}\right) + \frac{\pi}{3}}$

$\boxed{0.186\left(x - \frac{\pi}{6}\right) + \frac{\pi}{3}}$

b) $f(x) = \begin{cases} x^2 - 2, & x < 1 \\ \frac{1}{2}x - \frac{5}{2}, & x \geq 1 \end{cases}$ $\quad \lim_{x \to 1^-} f(x) = -1$, $\quad \lim_{x \to 1^+} f(x) = -2$ } no limit at $x = 1$ so no tangent line

2pts

(3) (a) $xy = \sin x + y^2 \rightarrow x\frac{dy}{dx} + y = \cos x + 2y\frac{dy}{dx}$

1pt

$\frac{dy}{dx}(x - 2y) = \cos x - y \qquad \boxed{\frac{dy}{dx} = \frac{\cos x - y}{(x-2y)}}$ or both reversed

(b) $y = \sin(x^3 - 5x + 1) \rightarrow \boxed{\cos(x^3 - 5x + 1)\cdot(3x^2 - 5)}$

(c) $y = (x^3 - 1)\cos x \rightarrow (x^3 - 1)\cdot -\sin x + \cos x \cdot 3x^2$

$$\boxed{\frac{dy}{dx} = 3x^2\cos x - (x^3 - 1)\sin x}$$

(d) $y = \frac{2x+5}{3x-1} \Rightarrow \frac{(3x-1)(2) - (2x+5)(3)}{(3x-1)^2} \Rightarrow \frac{\cancel{6x} - 2 - \cancel{6x} - 15}{(3x-1)^2}$

$$\boxed{\frac{dy}{dx} = \frac{-17}{(3x-1)^2}}$$

(4) $f'(x) = 4x^3 - 8x^2$

$0 = 4x^2(x-2)$

$f'(x) = 0$ at $0, 2$

$- \quad - \quad +$

$0 \quad 2$

decreasing increasing

2 → min

$f''(x) = 12x^2 - 16x$

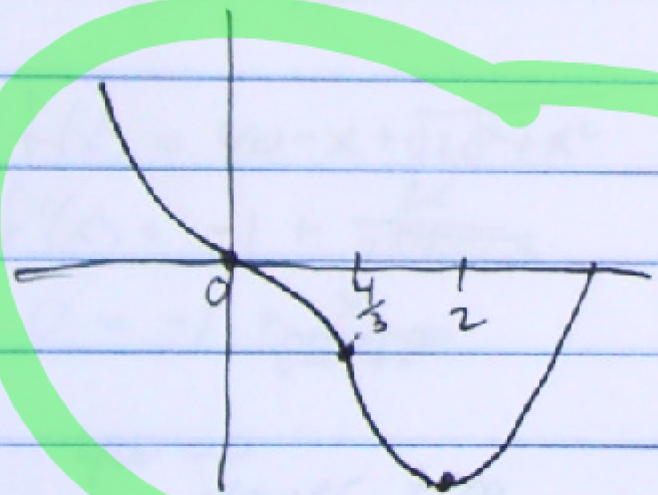
$0 = 4x(3x-4)$

$f''(x) = 0$ at $x = 0, \frac{4}{3}$

$+ \quad - \quad +$

up 0 down $\frac{4}{3}$ up

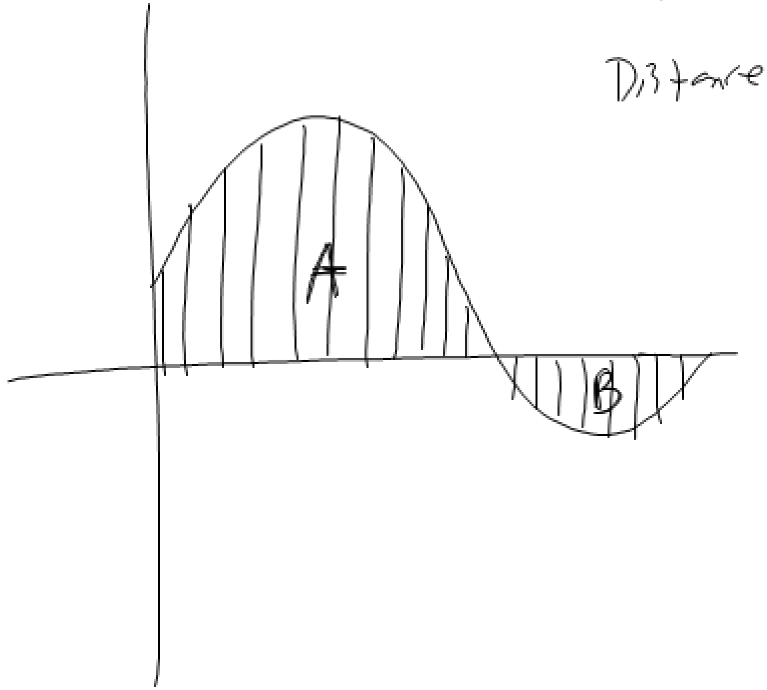
→ inflect. pt.

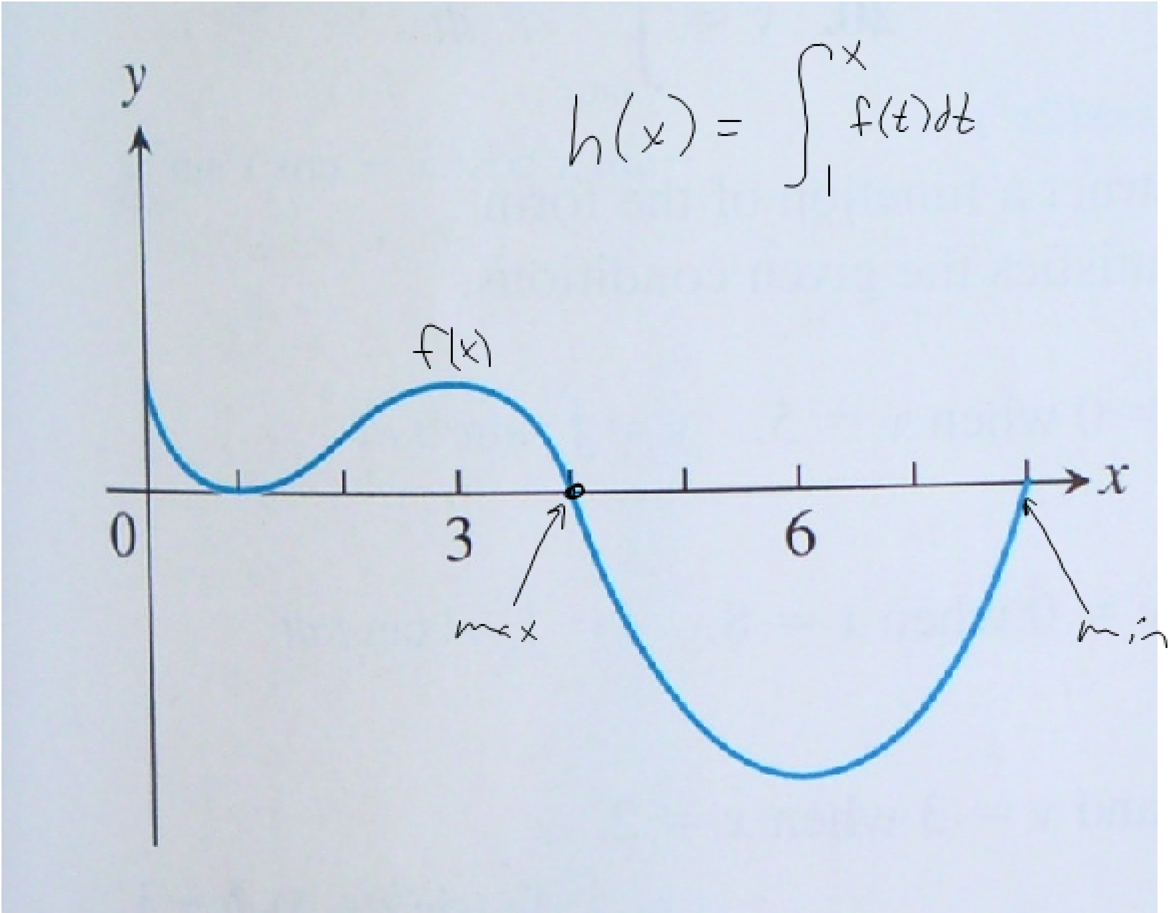


possible graph

Displacement → Net Area (subtracts the area B)

Distance → Total Area ($A + |B|$)





$$h(1) = 0$$

$$h(0) = \int_1^0 f(t)\,dt$$

$$= \text{negative}$$

Sect. 5.4

\# 41–48, 55, 57, 58, 60